are the relative frequencies or probabilities of occurrence of $x_1(t)$ alone, $x_2(t)$ alone, and $x_1(t)+x_2(t)$, respectively. Clearly, $1 = p_1+p_2+p_{1,2}$ is the probability of an event certain to occur.

In terms of cumulative frequencies or probabilities of occurrence, Eq. (28) reduces to

$$\bar{\mu}_x = \hat{p}_1 \bar{\mu}_{x_1} + \hat{p}_2 \bar{\mu}_{x_2} \tag{29}$$

where

$$\bar{\mu}_{x_1} = (p_1 {}^i\bar{\mu}_{x_1} + p_{1,2} {}^k\bar{\mu}_{x_1})/\hat{p}_1, \quad \bar{\mu}_{x_2} = (p_2 {}^j\bar{\mu}_{x_2} + p_{1,2} {}^k\bar{\mu}_{x_2})/\hat{p}_2$$

and where the time fractions $\hat{p}_1 = p_1+p_{1,2}$ and $\hat{p}_2 = p_2+p_{1,2}$ are the probabilities of occurrence of $x_1(t)$ or $x_1(t)+x_2(t)$ and $x_2(t)$ or $x_1(t)+x_2(t)$, respectively.

Further development of this case is left to the interested reader. It is important to note, however, that setting $p_{1,2}=0$ implies that $x_1(t)$ and $x_2(t)$ are mutually exclusive 100% of the time, and setting $p_{1,2}=1$ implies $x_1(t)$ and $x_2(t)$ act concurrently 100% of the time. The former case illustrates how to resolve $x(t)$ into mutually exclusive components, and the latter case leads to the principle of linear superposition. Finally, it should be noted that the p and $\hat{p}$ values are discrete and can also be determined by counting occurrences.

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On the Accuracy of the Taylor Approximation for Structure Resizing

OLAF O. STORAASLI* AND JAROSLAW SOBIESZCZANSKI†
NASA Langley Research Center, Hampton, Va.

Introduction

CONDUCTING full analyses each time a structure is modified (i.e., structural elements resized, material properties changed) makes the repetitive design process costly. Thus, several "short cut" structural modification algorithms[1–3] have been proposed. Their common drawback is that it becomes more economical to reanalyze the complete new structure rather than to use the modification method even when only a relatively small number of elements are modified. However, use of the Taylor expansion[4] for structural resizing is applicable to modifications involving a large number of structural members, ultimately even resizing all elements of a structure simultaneously. Since it is an approximate method, the potential it offers hinges on its accuracy. Therefore, results of a series of numerical experiments to test the method's accuracy were obtained for a large complex structure, the midsection of an idealized aircraft fuselage (Fig. 1). A more detailed description of the method and its application is found in Ref. 5.

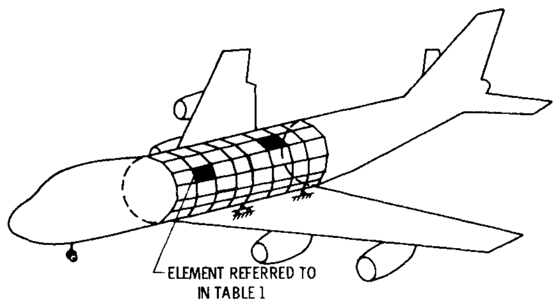

Fig. 1 Fuselage midsection.

Method

The pertinent equations are the familiar load $\{L\}$, deflection $\{u\}$, equation

$$[K]\{u\} = \{L\} \tag{1}$$

its derivative with respect to the design variable V_i

$$[K](\partial\{u\}/\partial V_i) = -(\partial[K]/\partial V_i)\{u\} \tag{2}$$

and the first-order Taylor expansion for the displacements

$$\{u\}_{\text{modified}} = \{u\}_{\text{original}} + (\partial\{u\}/\partial V_i)\cdot\Delta V_i \tag{3}$$

Note that the stiffness matrix, $[K]$, is a function of the design variables, V_i (i.e., thickness, area, or any stiffness property of individual finite element), however, $\{L\}$ is considered independent of V_i thus neglecting dead load.

Considering the formal similarity of Eqs. (1) and (2), one may obtain gradients, $\partial\{u\}/\partial V_i$ by repeating only the back substitution (at 1/39th the cost of reanalysis in this case) portion of the original solution of Eq. (1) with the right-hand side of Eq. (2) playing the role of a pseudo load.

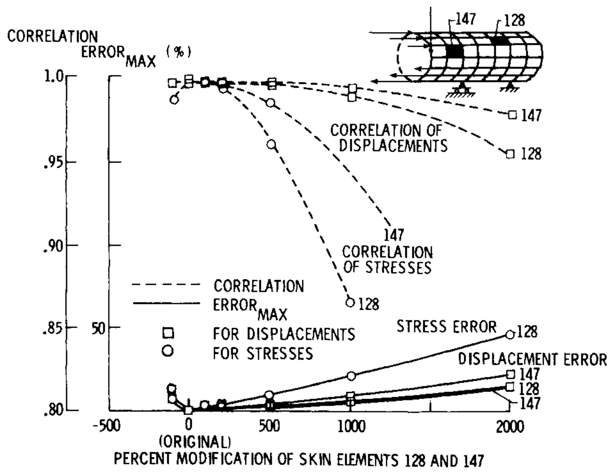

Fig. 2 Accuracy of Taylor expansion for resized skin elements.

Presented as Paper 73-338 at the AIAA/ASME/SAE 14th Structures, Structural Dynamics, and Materials Conference, Williamsburg, Va., March 20–22, 1973; submitted April 23, 1973; revision received September 25, 1973. The authors would like to acknowledge the fruitful discussions held with L. Schmit, UCLA, on the concepts and benefits of the Taylor expansion approach for improving structural synthesis methods.

Index categories: Structural Design, Optimal; Structural Static Analysis.

* Research Scientist, Design Methods Section. Member AIAA.

† Research Scientist, IPAD Technology Section. Associate Fellow AIAA.

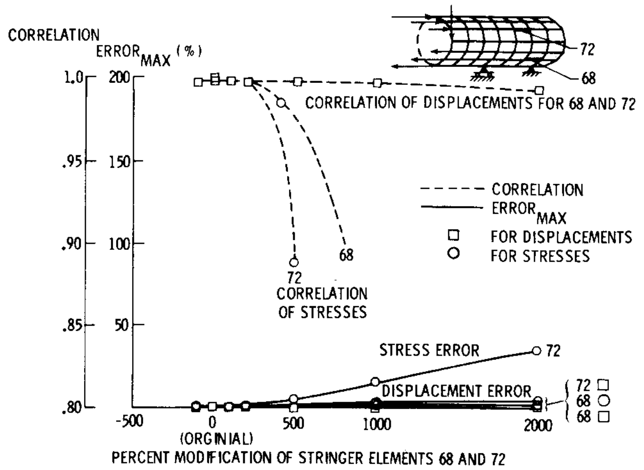

Fig. 3 Accuracy of Taylor expansion for resized stringer elements.

Table 1 Taylor approximation vs reanalysis

Thickness change, %	Displacement errors, %	Stress errors, %	Displ. correl. coeff.	Stress correl. coeff.
−100 (cutout)	−3.152	−0.7684	0.99974	0.99659
0 (original)	0.0	0.0	1.0 (identical)	1.0 (identical)
+100	−0.2689	−0.1571	0.99999	0.99979
+200	−0.9898	−0.4734	0.99994	0.99895
+500	−3.875	−2.276	0.99932	0.98789
+1000	−9.288	−5.798	0.99613	0.93437

Accuracy Evaluation

Accuracy was evaluated by comparing displacements and stresses obtained from the Taylor expansion with those obtained by a complete analysis. In all the numerical tests, a highly idealized model of a transport aircraft fuselage midsection, shown in Fig. 1, was used. It consists of quadrilateral membrane elements (skin), and bar elements (stringers and frames) and was loaded by forces simulating the bending and shear action of the aft portion of the fuselage while equilibrium was provided by supports representing wing spars. Initially all member thicknesses or cross-sectional properties were constant lengthwise and hoopwise.

Modifications ranged from −100% (element removal) to 2000% for a single element, and from −50% to 50% for multiple elements. The modifications consisted of alterations of skin thicknesses, cross-sectional areas of stringers, and simultaneous changes of area, and torsional and bending [cross-sectional] inertias for frames.

Stresses and displacements were sorted into 10 absolute magnitude groups, so-called deciles in statistical terminology, and a comparison was carried out for each group separately. This Note displays accuracy evaluation results for the highest decile as being of greatest practical interest. In addition, for an over-all accuracy measure, a statistical analysis was performed on the data from all deciles to obtain the standard correlation coefficients described in Ref. 6.

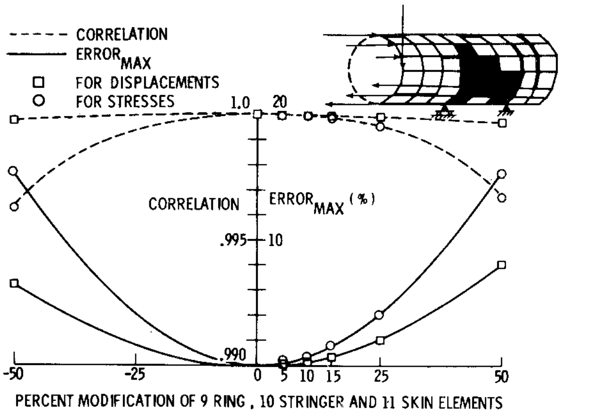

Fig. 5 Accuracy of Taylor expansion for simulated design (multiple elements resized).

Results

Table 1 and Figs. 2–6, show the method's accuracy by condensing point-by-point comparisons of over 5000 displacements and 15,000 stress components for three levels of element resizing: single, multiple, all.

The results shown in Table 1 for modification to the single panel, indicated in Fig. 1, reveal that a thickness change of up to 500% results in stress and displacement errors of less than 3% and 4%, respectively, for the highest decile, with over-all correlation of 0.98. Figure 2 shows the accuracy data for this element (147), compared with that of another skin element further aft (128). Shown along the abscissa is the range of modification of the element's thickness, and along the ordinate the accuracy represented by ERROR$_{MAX}$ and CORRELATION. Figures 3 and 4 show similar information for changes

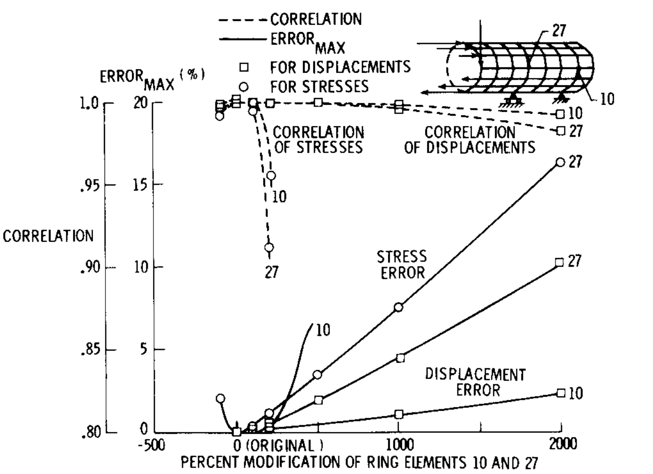

Fig. 4 Accuracy of Taylor expansion for resized frame elements.

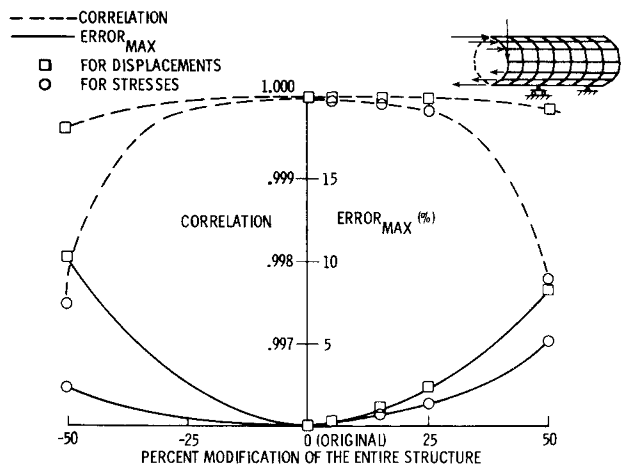

Fig. 6 Accuracy of Taylor expansion for entire structure (all elements resized).

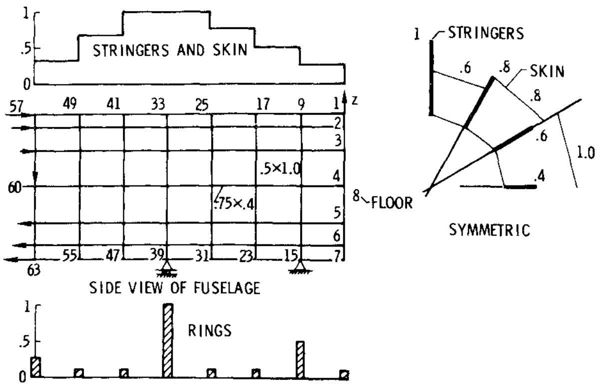

Fig. 7 Material redistribution for entire structure resizing [i.e., resized thickness of panel between nodes 11, 12, 20, 19 is proportional to 0.5 (from upper graph) times 1.0 (from the graph on the right)].

to stringer and frame (ring) elements, respectively. From these and other data collected, it appears that for single element modifications of up to 500%, the errors do not exceed 4% for displacements and 16% for stresses.

For simultaneous multiple element and entire structure modifications, the element stiffness properties were changed from the original values to simulate a typical "beefing up" operation, which usually follows the initial stress analysis in a design process. Figure 5 shows the accuracy of the method when the shaded elements are strengthened in proportion to their original stresses. For this simulated design, the errors were less than 8% for displacements and 16% for stresses over the modification magnitude interval −50% to +50%.

Extension of this multiple modification to involve the entire structure, yielded an error not exceeding 5% for stress and 10% for displacement in the same modification interval (see Fig. 6). This accuracy is better than in the previous case, probably due to smoother distribution of the added material, shown in Fig. 7. This distribution simulates a single complete step of a fully stressed design procedure in which each element is resized in proportion to its stress.

Conclusions

Investigation of the Taylor expansion accuracy, when applied as an approximation to a modified structure solution, was carried out using a highly idealized finite element representation of a skin-stringer-frame fuselage midsection as a test model. It was found that for the sample structure, this approximation's error is less than 16% over a range of −100% (element removal) to +500% for modification of a single element, and −50% to 50% for simultaneous multielement modifications.

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Crack in a Thin Infinite Plate of Viscoelastic Medium

SATYANARAYAN MANDAL*
Hatuganj M.N.K. High School, Hatuganj, Parganas, West Bengal, India

Introduction

A GOOD number of problems concerning the growth of cracks have been solved in classical elasticity theory. In this connection a recent book by Sneddon and Lowengrub[1] may be mentioned. In viscoelasticity theory the analysis of crack problems presents some difficulties which are absent in elasticity theory.[2]

It appears that the crack problems in a viscoelastic medium have been paid much less attention until recently. In this connection the papers[2–4] may be mentioned. Munshi[5] has discussed the crack problem in an infinite plate of anisotropic elastic medium with cubic symmetry. This problem deals with the crack opened by a constant normal pressure over a small segment inside the plate and has been solved by the complex potential method.

In this Note an attempt has been made to obtain the stress field and the components of the displacement vector due to the application of a pressure inside the crack contained in a thin viscoelastic plate of infinite extent. The viscoelastic material is of the general linear type. The prescribed internal pressure which varies along the length of the Griffith crack has been applied to open it. Considering the prescribed stress to be a uniform pressure, the shape of the crack is found to be an elliptic one at a particular instant of time.

Formulation of the Problem and Method of Solution

The stress-strain relation for the homogeneous viscoelastic medium of the general linear type is taken as

$$\left(1+a_1\frac{\partial}{\partial t}\right)\sigma_{ij} = 2k_1\left(1+b_1\frac{\partial}{\partial t}\right)e_{ij} \tag{1}$$

where a_1, b_1, k_1 are material constants.

Here it is assumed that the Poisson's ratio is zero. Under this assumption the stress-strain relation for the normal and the shearing components are of the same form.

Equations of equilibrium in the absence of the body force are

$$\left.\begin{aligned}(\partial\sigma_{xx}/\partial x)+(\partial\sigma_{xy}/\partial y) &= 0\\ (\partial\sigma_{xy}/\partial x)+(\partial\sigma_{yy}/\partial y) &= 0\end{aligned}\right\} \tag{2}$$

On $y = 0$, boundary conditions are given by

$$\begin{aligned}\sigma_{xy} &= 0 && \text{for all } x\\ \sigma_{yy} &= -p(x)H(t) && \text{for } |x|<1\\ U_y &= 0 && \text{for } |x|\geq 1\end{aligned} \tag{3}$$

Applying Laplace transform defined by

$$\bar{F}(x,y,S) = \int_0^\infty \bar{e}^{St} F(x,y,t)\,dt \tag{4}$$

to Eqs. (1) and (2) we have

$$\begin{aligned}(1+a_1S)\bar{\sigma}_{xx} &= 2k_1(1+b_1S)\bar{e}_{xx} = 2k_1(1+b_1S)\,\partial\bar{U}_x/\partial x\\ (1+a_1S)\bar{\sigma}_{yy} &= 2k_1(1+b_1S)\bar{e}_{yy} = 2k_1(1+b_1S)\,\partial\bar{U}_y/\partial y\\ (1+a_1S)\bar{\sigma}_{xy} &= 2k_1(1+b_1S)\bar{e}_{xy} = k_1(1+b_1S)\left(\frac{\partial\bar{U}_x}{\partial y}+\frac{\partial\bar{U}_y}{\partial x}\right)\end{aligned} \tag{5}$$

Received April 27, 1973; revision received June 28, 1973. The author is grateful to S. K. Sarkar, Department of Mathematics, Jadavpur University, for his guidance in the preparation of this Note.
Index categories: Aeroelasticity and Hydroelasticity; Aircraft Structural Materials.

* Teacher.